Dynamics of Reduced El Niño Models

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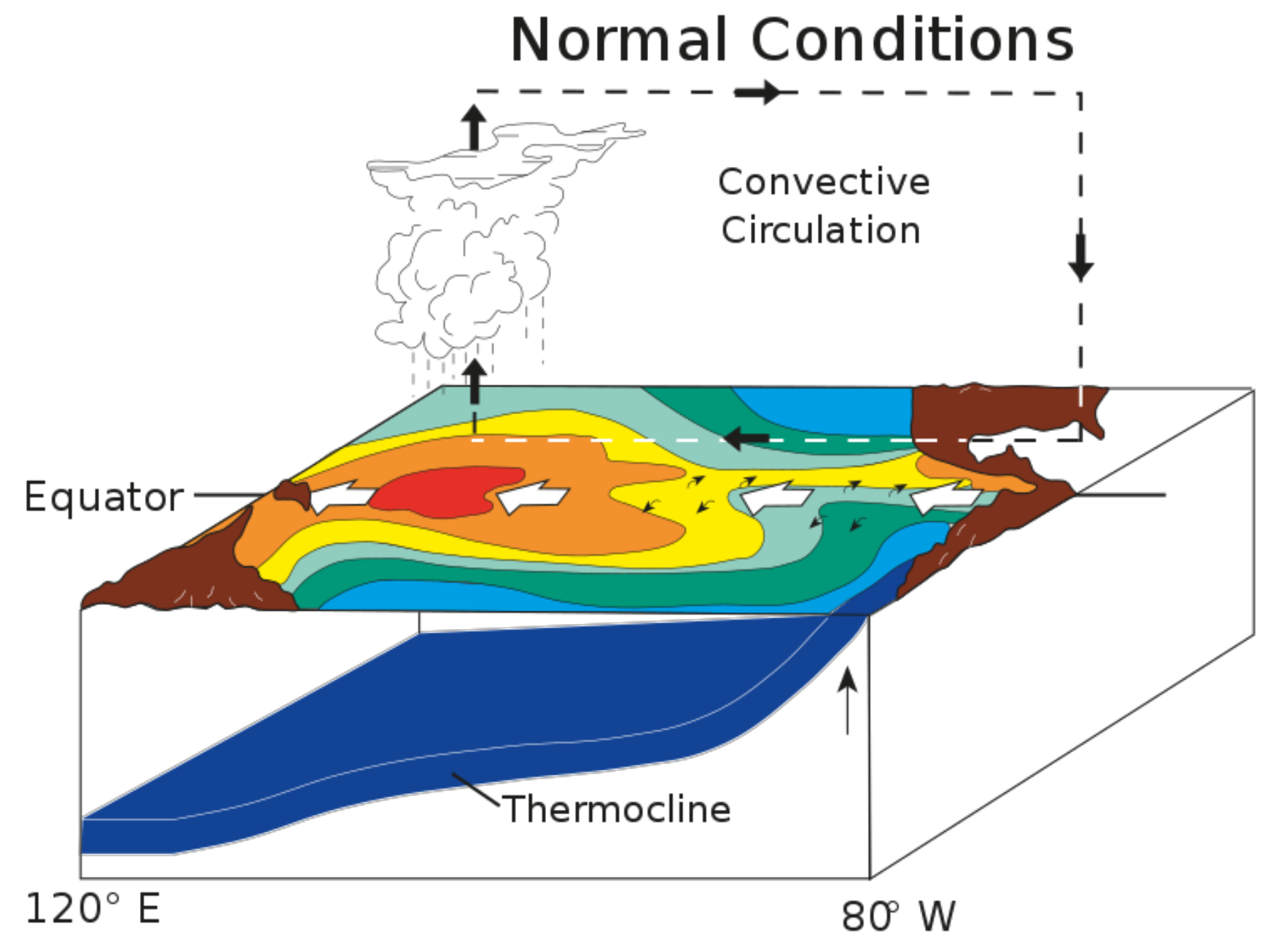
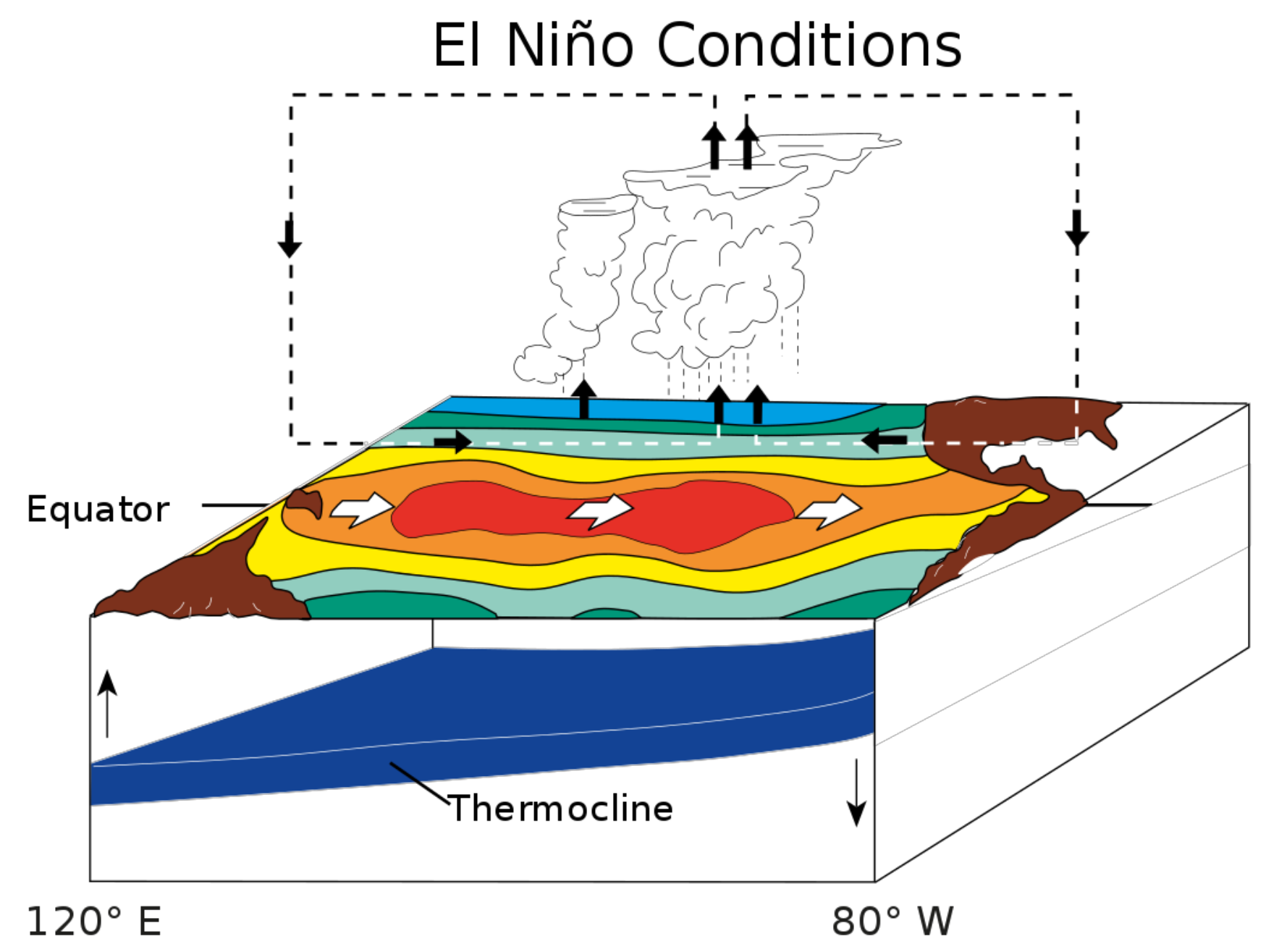
What is El Niño?

El Niño-Southern Oscillation is a typical extreme climatic phenomenon that fluctuates between El Niño, La Niña, and neutral states.

El Niño events, associated with higher sea surface temperatures in the Eastern Equatorial Pacific, are caused by atmospheric and oceanic interactions.





Modeling El Niño

The early stages of understanding and modeling El Niño are marked by the seminal papers of Bjerknes, Wyrtki, and Cane & Zebiak.

Cane and Zebiak [1] built a dynamical ENSO model in 1985 and successfully tested the hypotheses of Bjerknes and Wyrtki. Cane-Zebiak model is the first coupled oceanic-atmospheric model of El Niño. It is a partial differential equation model based on shallow water equations.

A reduced El Niño model: Recharge discharge oscillator

Timmermann, et al. [2] proposed a reduced, nonlinear conceptual model of El Niño in 2003. This model develops the recharge discharge oscillator model and can capture the El Niño bursting behavior.

$$\frac{dT_1}{dt} = -\alpha(T_1 - T_r) - \frac{u(T_2 - T_1)}{(L/2)}$$

$$\frac{dT_2}{dt} = -\alpha(T_2 - T_r) - \frac{w(T_2 - T_{\text{sub}})}{H_m}$$

$$\frac{dh_1}{dt} = r\left(-h_1 - \frac{bL\tau}{2}\right)$$

where

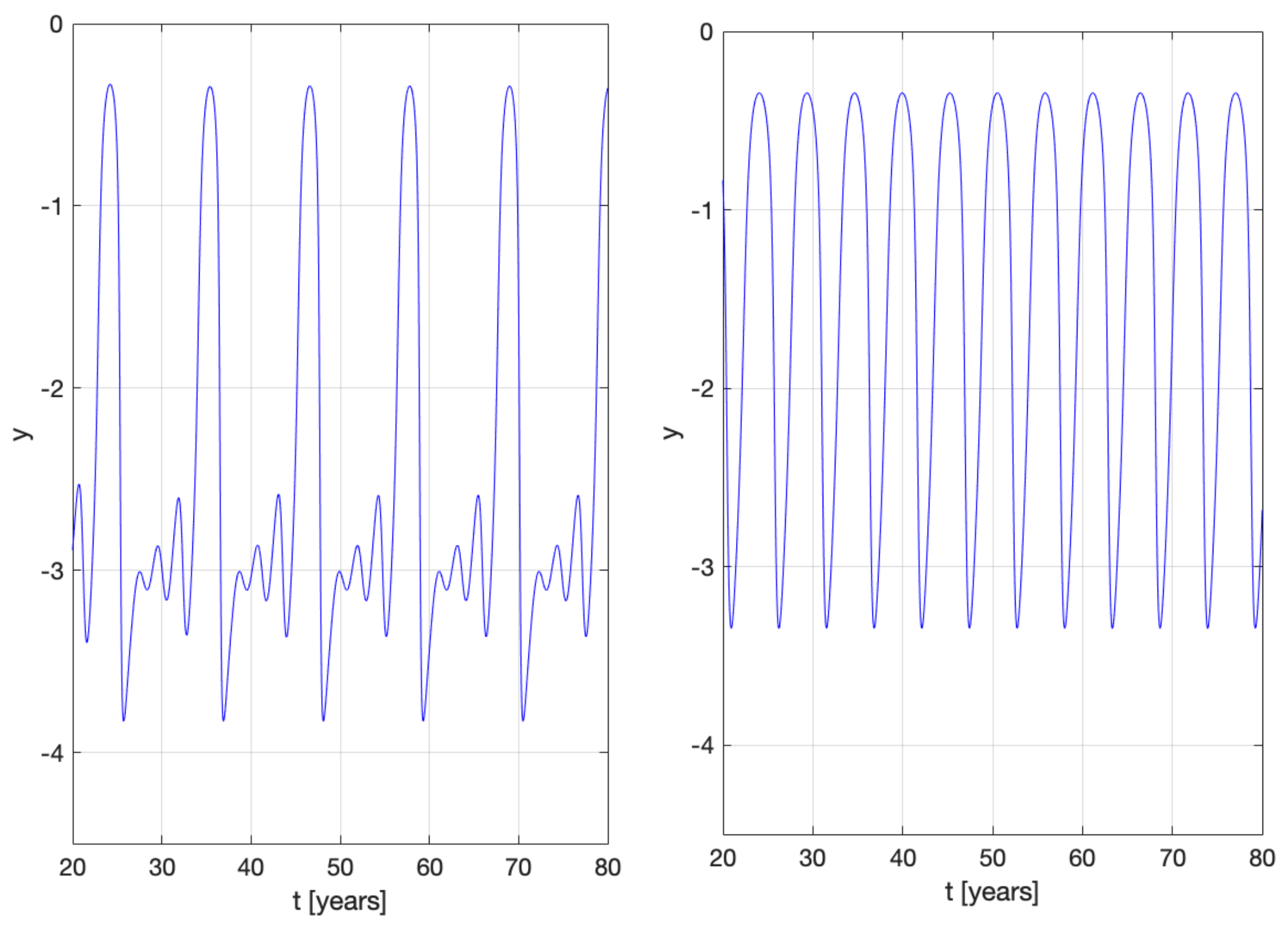
$$T_{\text{sub}}(T_1, T_2, h_1) = \frac{T_r + T_{r_0}}{2} - \frac{T_r - T_{r_0}}{2}\tanh\left[\frac{H - z_0 + h_1 + bL\mu(T_2 - T_1)/\beta}{h^*}\right].$$

Nondimensionalization

By substitution, change the above Timmermann's model into

$$\begin{cases} \dot{x} &= -\eta x - \lambda x^2 + \nu x[y + c\,(1 - \tanh(z - x))] \\ \dot{y} &= -\eta y - \nu x[y + c\,(1 - \tanh(z - x))] \\ \dot{z} &= -\varphi\left(z - k - \dfrac{x}{2}\right) \end{cases}$$

Time series plots



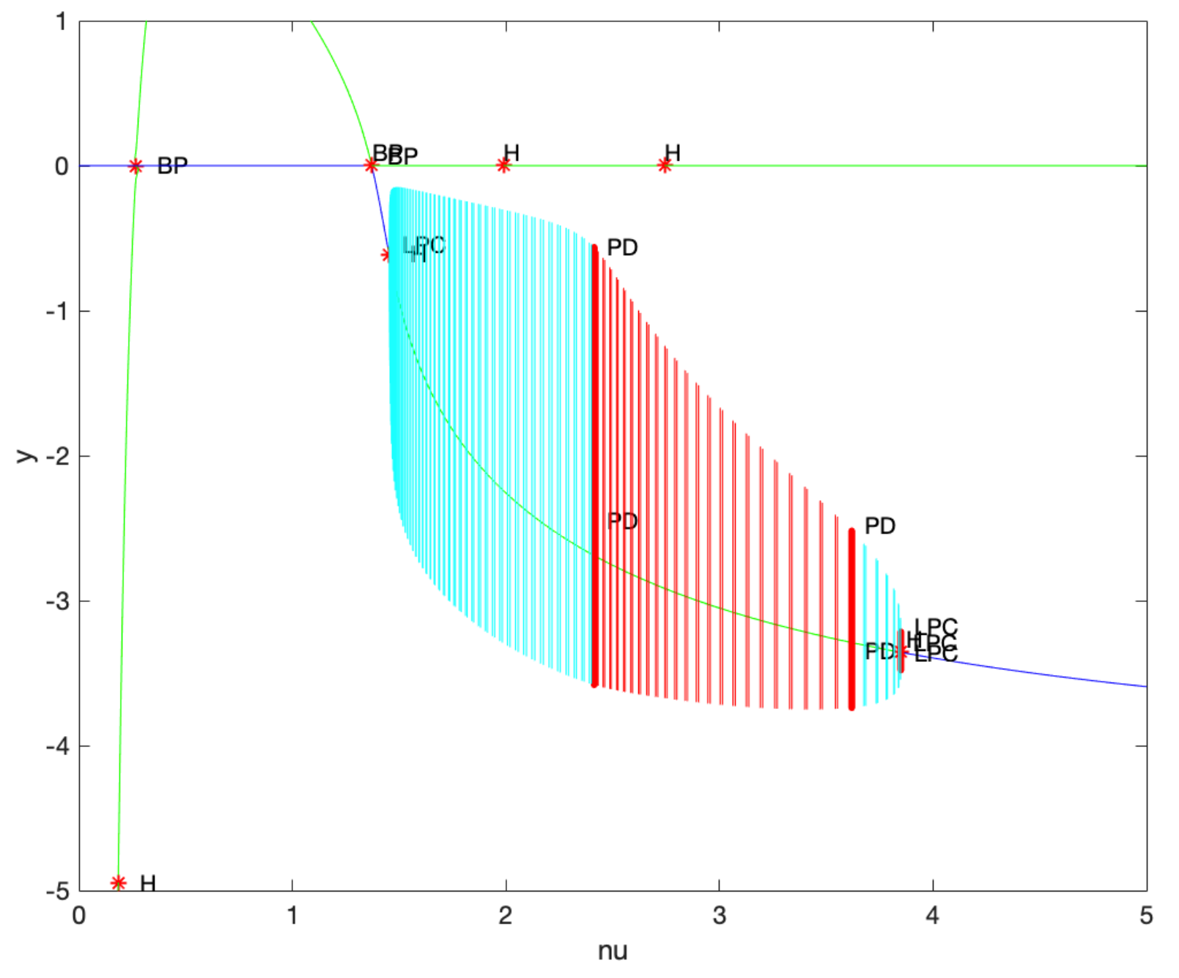
λ=0.2674, ν=2.8970 λ=0.3209, ν=2.1396

The dimensionless model could exhibit the bursting behavior characterized by decadal occurrences of strong El Niño events, shown in the left plot. Under its specific parameter settings, the system can generate amplitude modulations and reproduce some qualitative similarities of observations.

The right plot shows a regular oscillation mode of El Niño with a different parameter settings. The period of oscillation is about 4~5 years. In fact, the period of ENSO is about 3~4 years. Therefore, this model captures the ENSO processes and bursting behaviors.

Bifurcation analysis

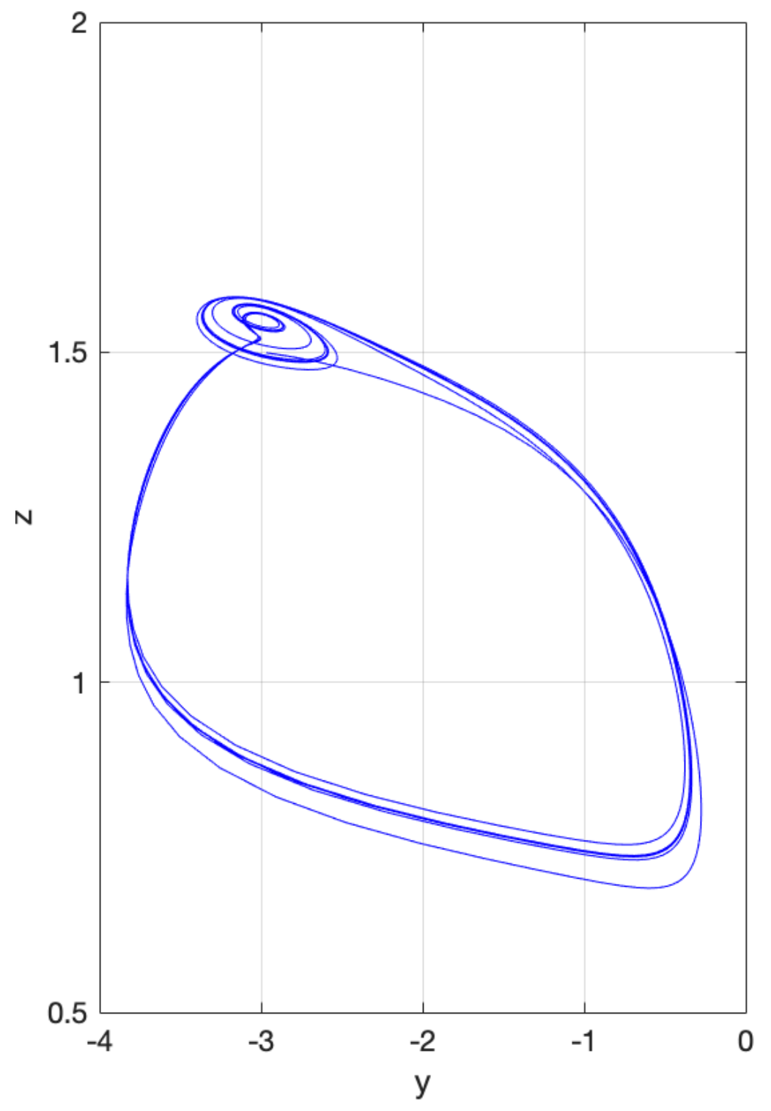
To study the bifurcation structure of the dimensionless model, we perform a bifurcation analysis where ν is used as parameters. The analysis is done in two steps: (1) continuation of equilibrium points, detection of Hopf bifurcations; (2) continuation of the periodic solutions.



Fix λ=0.2674, the above diagram of y over ν displays the stability changes.

- Blue curve means stable fixed point, green means unstable fixed point, cyan means stable limit cycle, red means unstable limit cycle.
- At ν=2.4135, a periodic doubling bifurcation gives rise to the amplitude modulated oscillatory regime of El Niño.

Phase space plot



For 2.4135<ν<3.6193, the trajectories spiral outwards, gain amplitude, move close to the constant zero equilibrium, and then get repelled from it. Finally, the trajectories return back to the vicinity of the saddle focus and continue spiraling. This regime is also illustrated in the left time series plots.

[1] Cane, M. A., & Zebiak, S. E. (1985). A theory for El Niño and the Southern Oscillation. *Science*, 228(4703), 1085–1087.
[2] Timmermann, A., Jin, F.-F., & Abshagen, J. (2003). A nonlinear theory for El Niño bursting. *Journal of the Atmospheric Sciences*, 60(1), 152–165.