The Tail of Two Distributions

EXPLORING HOW THE VELOCITY DISTRIBUTION FUNCTIONS OF WARM AND COLD DARK MATTER AFFECT THE SHAPE OF THEIR RESPECTIVE MATTER POWER SPECTRA

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In this project, we employ the Cosmic Linear Anisotropy Solving System (CLASS)[2] to investigate how the velocity distribution functions of warm dark matter (WDM) and cold dark matter (CDM) affect the shape and location of the cut-off in their respective matter power spectra.

MOTIVATION

> Dark matter, or particles that do not interact with light, constitutes around 25% of the energy density of the universe[1].

> Dark matter candidates can be classified by their velocity dispersion.

> The velocity distribution of dark matter depends on when it decoupled from the Standard Model. WDM decoupled while relativistic, whereas CDM decoupled while non-relativistic.

> These distributions can determine the shape of the cut-off in the matter power spectrum, which sets the minimum size of dark matter halos and therefore the onset of early structure formation in the universe.

DEFINITIONS

> Matter Power Spectrum (P(k)): a measure of the density contrast of the universe as a function of scale.

> Transfer Function T(k) :

$$T(k) = \sqrt{\frac{P_{NCDM}(k)}{P_{\Lambda CDM}(k)}}$$

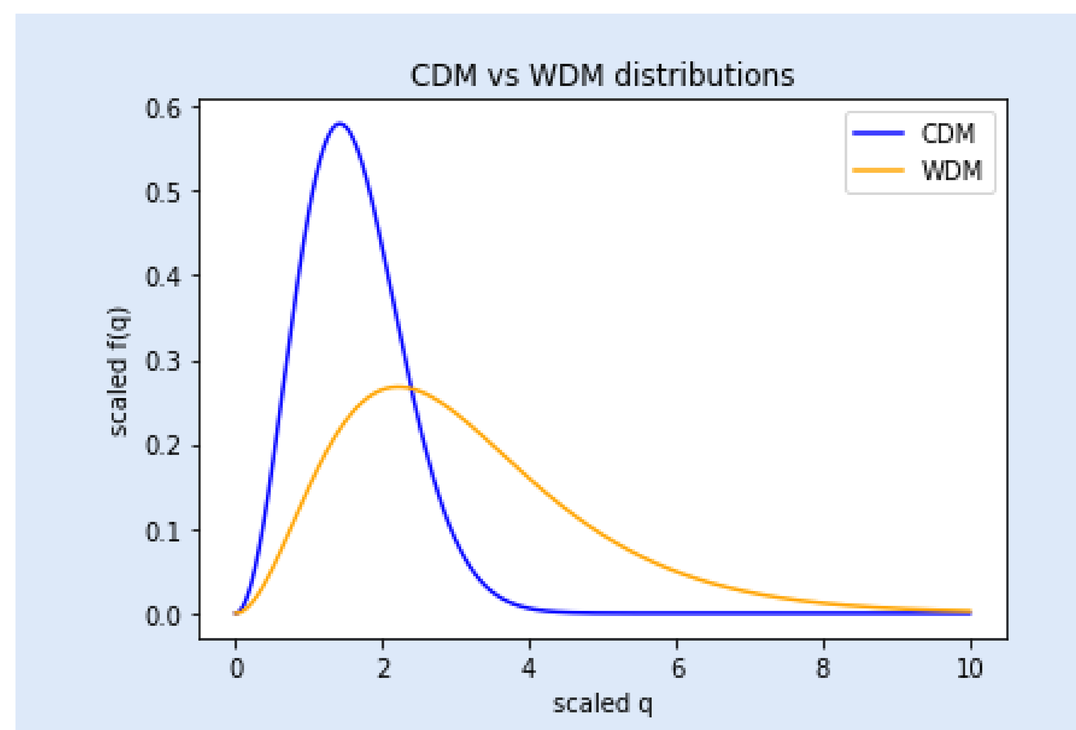


Fig 1: Two typical momentum distributions for WDM and CDM. Compared to CDM, WDM peaks at higher momentum ("q") values and has a hotter tail.

DISTRIBUTION FUNCTIONS

> To input velocity distribution functions into CLASS, you must recast them as momentum distributions which peak ~1 to avoid numerical errors (Fig 1).

> This gives you a 3-D distribution function f(q), where "q" is the comoving momentum of a typical particle scaled by its momentum today.

> You must provide CLASS with T_{NCDM}, the momentum of a typical particle today, scaled by the photon temperature $T_{PHOT} \sim 2.3*10^{-4}$, and M_{NCDM}, the mass of that particle.

STANDARD MODELS OF T(K)

$$T(k)_{WDM} = (1 + (k/k_{half})^{2v})(2^{v/5} - 1)^{-5/v}, v = 1.12$$

$$T(k)_{CDM} = e^{-(k/k_{half})^2 ln(2)}$$

where k_{HALF} is the value at which T(k) = 1/2.

FINDINGS

> K_{HALF}, the value where the transfer function is equal to 1/2, is proportional to the square root of M / T_{NCDM} (Fig 2).[3] The free streaming scale is proportional to K_{HALF}, and therefore the cut-off in the matter power spectrum is proportional to M / T_{NCDM} for the same distribution function.

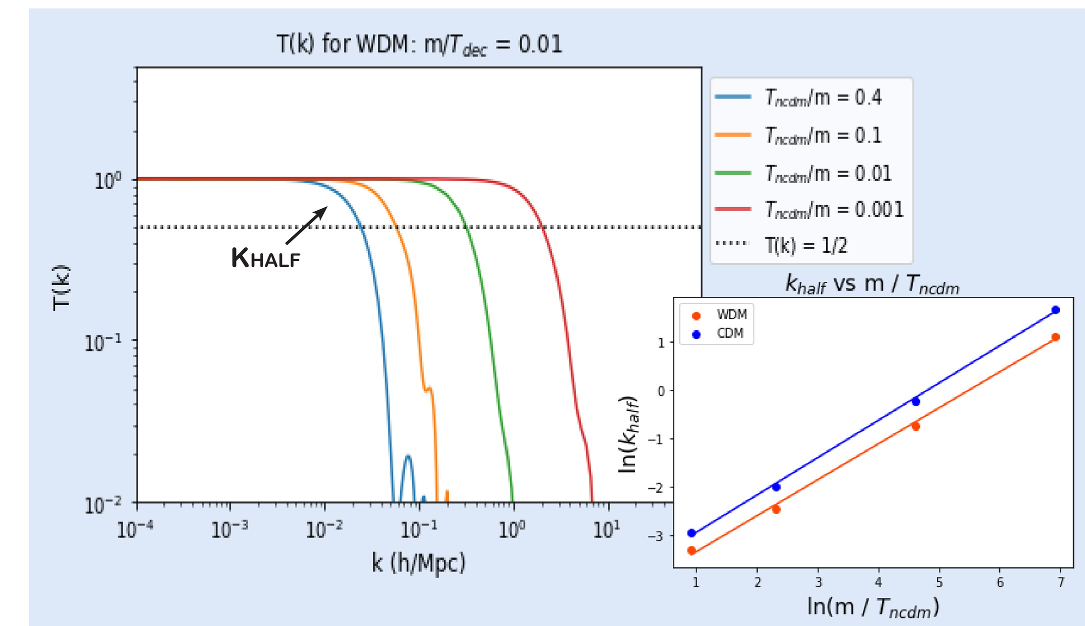


Fig 2: As T_{NCDM} / M is increased, the cut-off in the matter power spectrum shifts to lower k values.

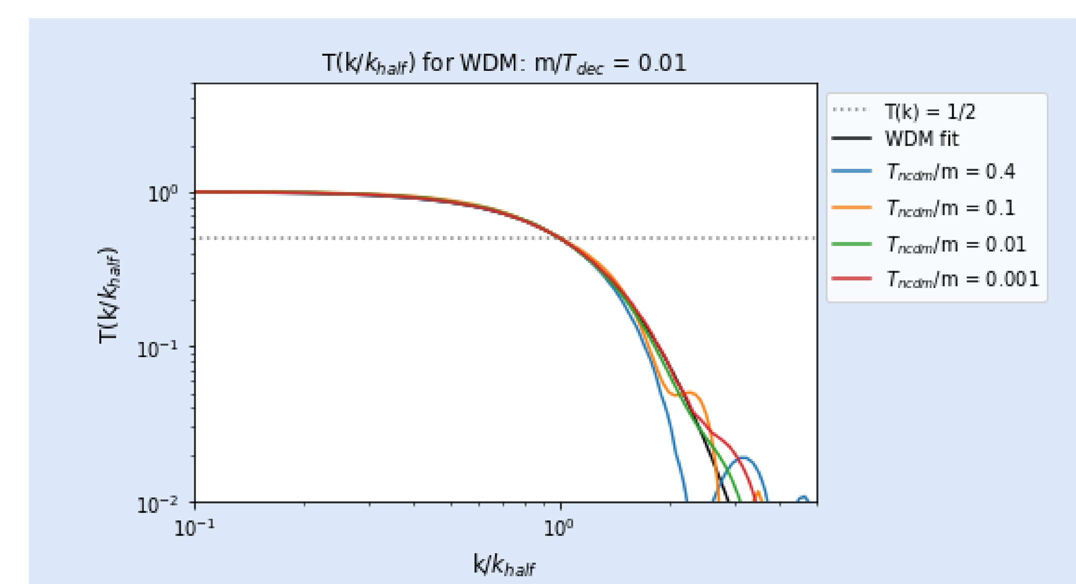


Fig 3: Transfer functions for WDM as the momentum of a typical particle is increased, compared to the standard model.

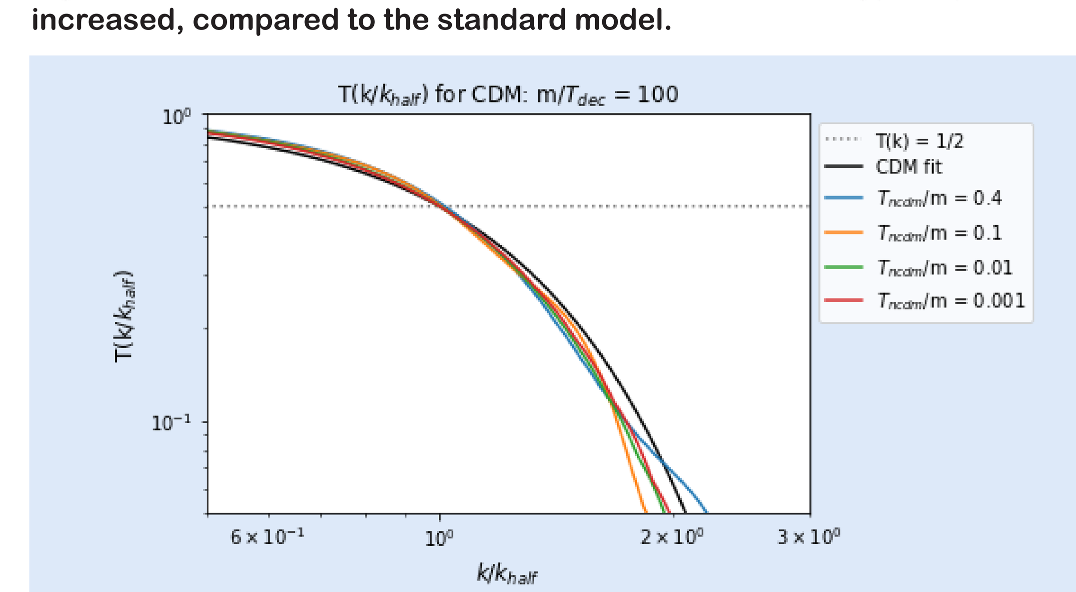
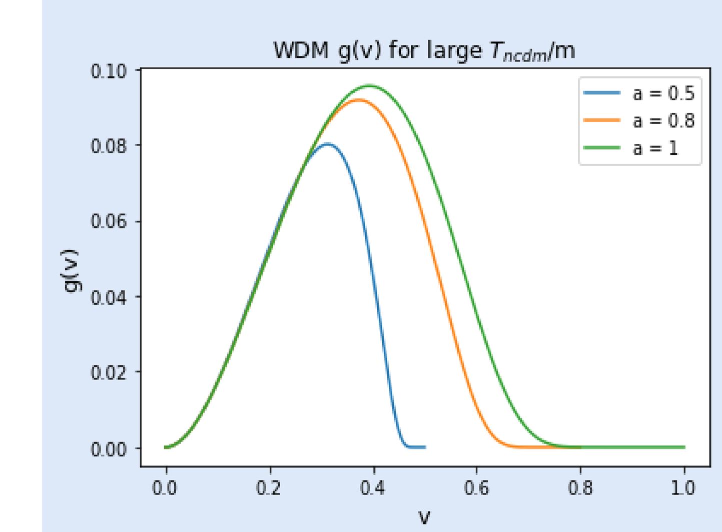


Fig 4: Transfer functions for CDM as momentum of a typical particle is increased, compared to the standard model.

> WDM starts deviating from the standardized model more quickly than CDM. This is due to WDM particles becoming relativistic more quickly than CDM particles, which causes CLASS to incorrectly map from a velocity to momentum distribution (Fig 3).

> CDM still experiences shape changes in its matter power spectrum and its cut-off, but they are less pronounced than those of WDM at the same T_{NCDM} / M ratio (Fig 4).

> We can translate the f(q) momentum distribution into g(v), the velocity distribution with relativistic factors taken into account. We find that g(v) is not equivalent to f(q) for relativistic particles since the velocity distribution becomes time-dependent. The shape of g(v) is clearly altered by increasingly relativistic particles at large T_{NCDM} / M which causes CLASS to incorrectly map onto momentum space (Fig 5).



Increasingly relativistic particles are not cooling quickly enough, g(v) changes, and therefore the shape of the matter power spectrum changes.

Non-relativistic particles produce the same velocity distribution function g(v) and therefore the same matter power spectra.

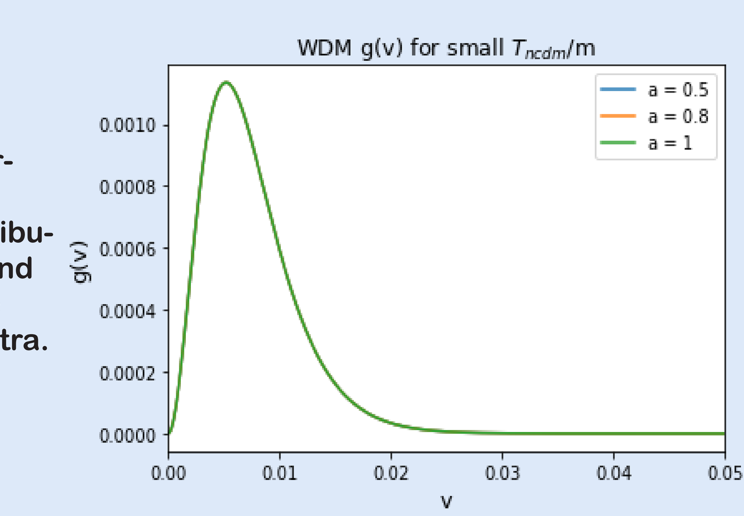


Fig 5: g(v), the velocity distribution of dark matter particles (not given to CLASS). Note that *v* here is actually velocity multiplied by the scale factor, *a*.

SUMMARY

> As the momentum of a typical particle is increased and the distribution function is kept constant, the transfer functions of WDM and CDM power spectra become increasingly inconsistent with their standardized models.

> Increasingly relativistic dark matter particles cause the velocity distribution function to become time-dependent (Fig 5) and in turn cause the shape of the cut-off in the matter power spectrum to change.

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3. Green, A. M., Hofmann, S., & Schwarz, D. J. (2004). The power spectrum of SUSY-CDM on subgalactic scales. Monthly Notices of the Royal Astronomical Society, 353(3), L23–L27. https://doi.org/10.1111/j.1365-2966.2004.08232.x