



# Numerical Methods for Learning Physical Constraints and their Applications

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## Introduction

Learning to predict physical phenomena poses many challenges and is computationally demanding, for:

- Sparse observations from real-world can be stochastic and chaotic.
- Physics priors are not available.
- Object information is not available.

There have been several early attempts to build DNNs that can learn physics properties (i.e. joints, lengths, angles, etc.) from limited sparse observations[1, 2, 3]. There are also discussions on learning physical properties from videos and reconstructing the videos from graphical simulations. Our method focuses on interpolation and extrapolation of the trajectory in simulations and videos and is easily pluggable into the existing frameworks.

Our contribution lies on:

- Improving prediction in chaotic environments by 100x.
- Achieving similar performance in regular environments.

## Theory

We propose a novel deep learning framework that takes sparsely sampled coordinates information and output interpolated or extrapolated coordinates.

First, we assume that the physical system we are observing can be described by a mass-spring-damper model. See the figure below for force analysis. Then, we show that the forces can be computed by

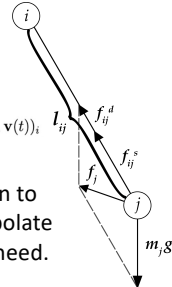
$$\vec{f}_i^s = k_s(|\vec{l}_{ij}| - l_0)\vec{n}_{ij} = k_s(|\vec{x}_j - \vec{x}_i| - l_0) \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|}$$

$$\vec{f}_i^d = k_d(\vec{v}_{ij} \cdot \vec{n}_{ij})\vec{n}_{ij} = k_d \left( (\vec{v}_j - \vec{v}_i) \cdot \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|} \right) \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|}$$

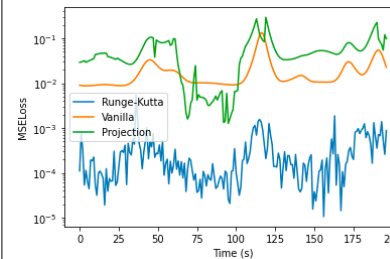
Hence, we conclude that forces depends only on coordinates and velocities.

$$(\mathbf{x}_i)_{t+1} = (\mathbf{x}_i)_t + \Delta t \cdot \text{net}(\mathbf{x}(t), \mathbf{v}(t))_i$$

Then, we can use numerical integration to interpolate or extrapolate the coordinates we need.



## Experiments



We test our model on two different scenes using MLP to approximate the net function -- double pendulum and rectangular boxes collision. We compare three different methods in such scenes:

- Our model with Runge-Kutta method.
- Vanilla method, which is bilateral interpolation.
- Projection Net[3].

We show that our method improved the Mean Square Error by 100x.

In the box collision experiment, we have comparable performance with bilateral interpolation.

## Conclusions

We propose a novel physics-aware method that enable coordinates interpolation and extrapolation. By doing so, we open the machine learning black box and add physics-aware part to it. Compared to previous attempts, we improve the performance in chaotic motions by 100x in terms of MSE and maintain the performance in general motions.

In the future, more physics-aware components can be added to this model, such as collision-detection, deformation, and other numerical methods.

### References

1. Greydanus, S., Dzamba, M., and Yosinski, J. Hamiltonian neural networks, 2019.
2. Hoshen, Y. Vain: Attentional multi-agent predictive modeling. arXiv preprint arXiv:1706.06122 (2017).
3. Yang, S., He, X., and Zhu, B. Learning physical constraints with neural projections, 2020.

Our model have several applications in sports analysis, video super-sampling, visual effects, etc. However, there are several challenges that need to be addressed:

- Motion blurs in images confuse object detectors.
- Object tracking. When objects collide, label may switch.

If we are can solve these two challenges, we can plug in our model in the existing object detection framework to denote and synthesize intermediate or future frames.

