Fast Haar Transformation in Dependency Test

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Compared with other basis choices, the non-standard Haar wavelet basis offers optimal local and sparse properties. It is naturally associated with an adaptive hierarchical tree structure, making it an ideal choice for analyzing high-dimensional data sets located on a low-dimensional manifold. However, to compute a number K of Haar wavelet expansion coefficients from N data points, a direct numerical method would require an expensive O(K*N) operations. In this research, we present a fast Haar Transform with asymptotically optimal complexity O(N+K). The new algorithm uses a divide-and-conquer strategy and processes the data efficiently on a hierarchical tree structure. The algorithm collects information directly from the data for the leaf nodes and stores the information in a compressed form. For each parent node at coarser levels, information is only collected from its four children nodes and the collected information is further compressed. The compression and recycling strategies make the fast Haar transform algorithm approximately K times faster than the direct method. Numerical simulation results are presented to demonstrate the algorithm's efficiency, accuracy, and asymptotic complexity. The developed Fast Haar Transform package is a powerful tool in the frequency domain-based statistical analysis in high dimensions.