Dark Matter-Lepton Interactions

Emma Modesitt, Advisor Dr. Adrienne Erickcek

In this project, we follow the popular theory of an early matter-dominated era (EMDE) that takes place after inflation but before the radiation-dominated era. During this time, we derive the final momentum distribution of collisions between relativistic leptons and non-relativistic dark matter particles.

MOTIVATION

- Within an EMDE, the dominant particle during this time decays into radiation – the relativistic lepton. These leptons then interact with dark matter particles, resulting in the particles having similar temperatures as defined by their momentum distributions.
- Once the interactions between the two particles decreases - a phenomenon called decoupling - the temperatures of the two particles diverge.
- The decoupling processes during an EMDE are complicated due to the presence of the relativistic leptons, which requires us to numerically simulation individual dark matter interactions.
- I am revisiting the scattering rate - derived from the time of decoupling - of the dark matter particles to find their final velocity distribution. This will help predict inhomogeneities within the structure of the Universe.

DEFINITIONS

Collision Term: C, the distribution, f, of dark matter momenta is determined by the elastic scattering of the particles, X.

Collision Rate: Γ, the rate at which collisions between the dark matter particles and leptons occur.

Momentum Transfer rate: γ, the rate at which momentum is transferred between the two particles.

RELATIVISTIC COLLISION

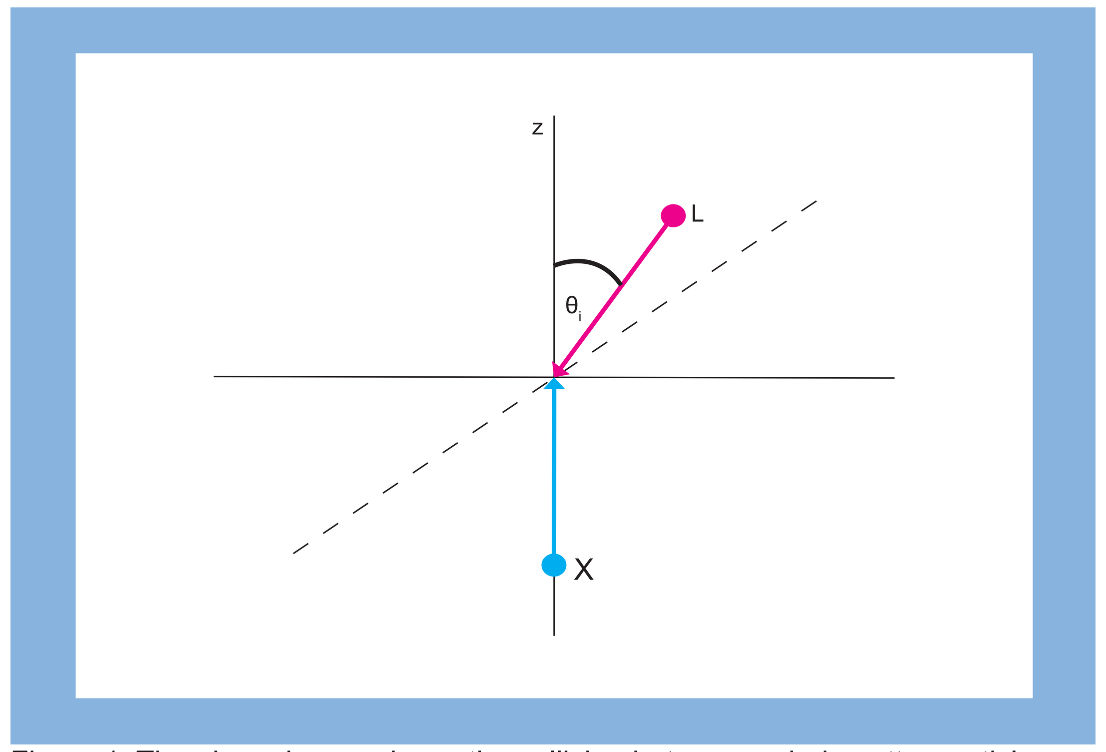

Figure 1. The above image shows the collision between a dark matter particle, X, and a lepton, L, in the comoving frame.

- The collision occurs in comoving space, where the dark matter particle moves along the +z axis and the lepton collides at an incidence angle θ_i.
- We then use a Lorentz boost to shift the collision into a frame where the dark matter particle is at rest. Another boost shifts into the center of momentum (COM) frame - this system has zero total momentum - and the momentum and energy of the particles can be easily calculated.
- Once the final momenta are found, we reverse the previous boosts and return to the comoving frame.

PARAMETERS

- Initial and scattering angles: θ_i [0,2π), θ_{CM} [0,π), and φ_{CM} [0,2π)
- Masses: m_X and m_L
- Initial momenta: p^o and k^o

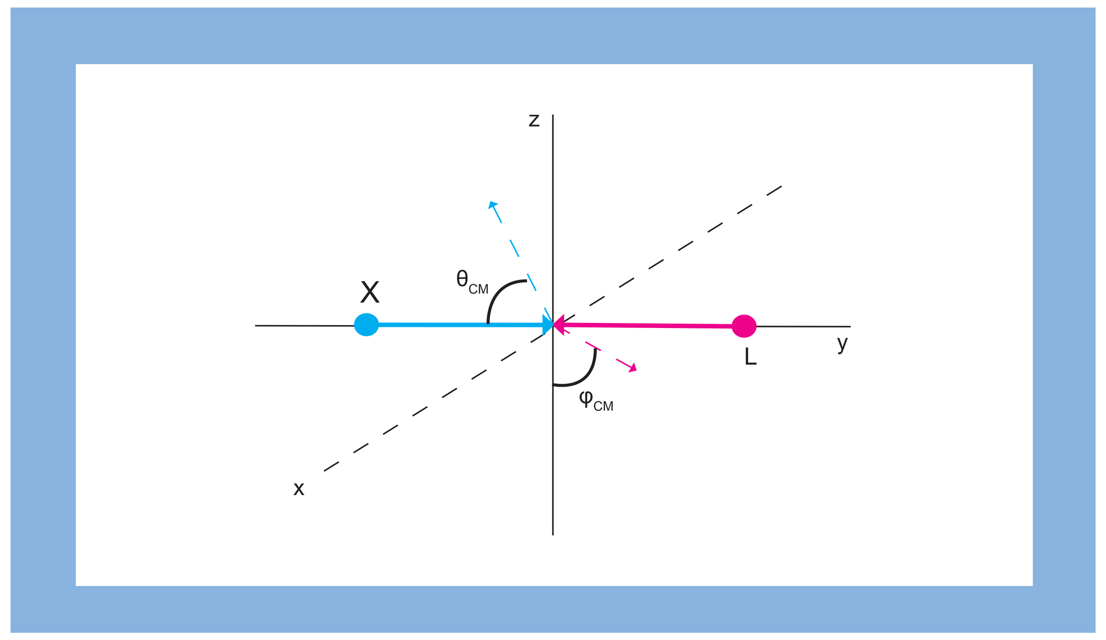

Figure 2. The diagram above shows the collision and resultant scattering between the dark matter particle and the lepton in the COM frame.

FINDINGS

- The final 4-momenta of the dark matter particle and lepton were derived and a simple code was constructed to verify conservation of energy and momentum.
- Compared to previous work, there were some differences in signage and constants.

$$\tilde{p}^{o\nu} = \begin{bmatrix} m_X\left[\gamma({\gamma_X^*}^2 - ({\gamma_X^*}^2-1)\cos\theta_{CM}) + (\sqrt{\gamma^2-1}\cos\theta)(\gamma_X^*\sqrt{{\gamma_X^*}^2-1}(1-\cos\theta_{CM})) - (\sqrt{\gamma^2-1}\sin\theta)(\sqrt{{\gamma_X^*}^2-1}\cos\phi_{CM}\sin\theta_{CM})\right] \\ m_X\sqrt{{\gamma_X^*}^2-1}\sin\phi_{CM}\sin\theta_{CM} \\ m_X\left[\sqrt{{\gamma_X^*}^2-1}(-\gamma_X^*\sin\theta(1-\cos\theta_{CM}) - (\cos\theta\cos\phi_{CM}\sin\theta_{CM}))\right] \\ m_X\left[-\sqrt{\gamma^2-1}({\gamma_X^*}^2 - ({\gamma_X^*}^2-1)\cos\theta_{CM}) - ((\gamma\cos\theta)({\gamma_X^*}^2\sqrt{{\gamma_X^*}^2-1}(1-\cos\theta_{CM}))) + ((\gamma\sin\theta)(\sqrt{{\gamma_X^*}^2-1}\cos\phi_{CM}\sin\theta_{\theta_{CM}}))\right] \end{bmatrix}$$

$$\tilde{k}^{o\nu} = \begin{bmatrix} \gamma\left[m_L\gamma_L^*\gamma_X^* + m_L\sqrt{{\gamma_L^*}^2-1}\cos\theta_{CM}\sqrt{{\gamma_X^*}^2-1}\right] + \sqrt{\gamma^2-1}\cos\theta\left[m_L\gamma_L^*\sqrt{{\gamma_X^*}^2-1} + \gamma_X^* m_L\sqrt{{\gamma_L^*}^2-1}\cos\theta_{CM}\right] + \sin\theta\sqrt{\gamma^2-1}\left[m_L\sqrt{{\gamma_L^*}^2-1}\cos\phi_{CM}\sin\theta_{CM}\right] \\ m_L\sqrt{{\gamma_L^*}^2-1}\sin\phi_{CM}\sin\theta_{CM} \\ m_X\left[-\sin\theta\sqrt{{\gamma_X^*}^2-1}(\gamma_L^* + \gamma_X^* m_X/m_L\cos\theta_{CM}) + \cos\theta\cos\phi_{CM}\sin\theta_{CM}\sqrt{{\gamma_L^*}^2-1}\right] \\ -\sqrt{\gamma^2-1}\left[m_L\gamma_L^*\gamma_X^* + m_L\sqrt{{\gamma_L^*}^2-1}\cos\theta_{CM}\sqrt{{\gamma_X^*}^2-1}\right] - \gamma\cos\theta\left[m_L\gamma_L^*\sqrt{{\gamma_X^*}^2-1} + \gamma_X^* m_L\sqrt{{\gamma_L^*}^2-1}\cos\theta_{CM}\right] - \gamma\sin\theta\left[m_L\sqrt{{\gamma_L^*}^2-1}\cos\phi_{CM}\sin\theta_{CM}\right] \end{bmatrix}$$

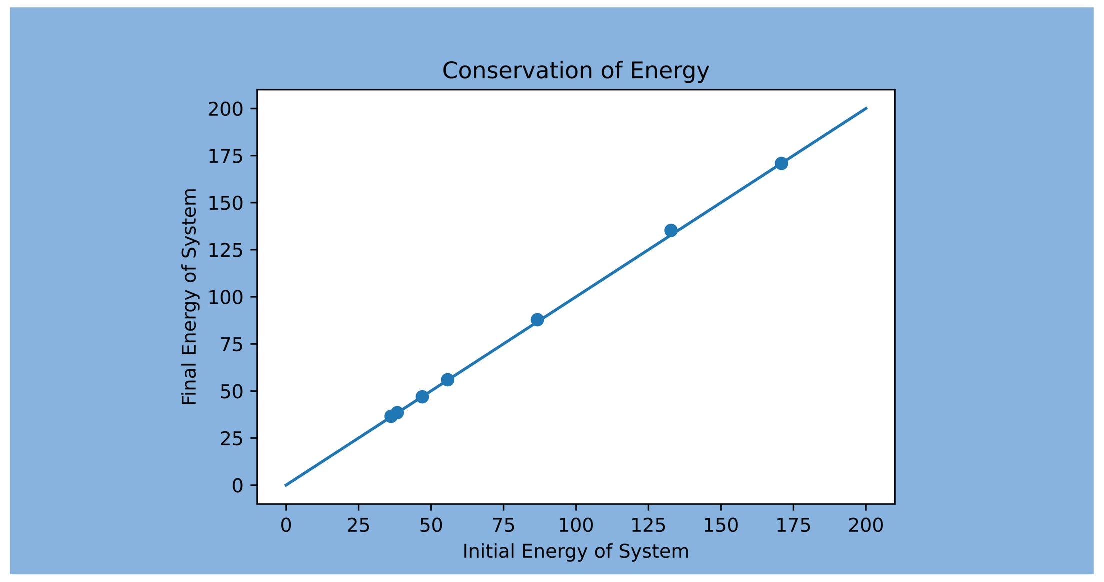

Figure 3. For arbitrary initial values of the parameters, the energy of the 2-particle system is equal in magnitude before and after the collision, demonstrating conservation of energy.

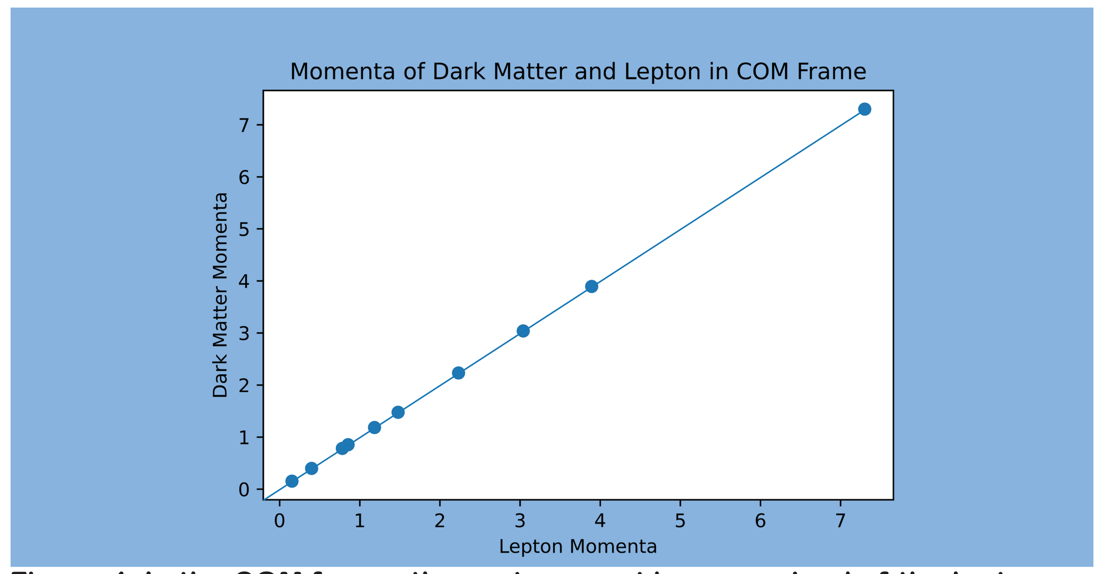

Figure 4. In the COM frame, the system must be comprised of the lepton and dark matter particles having equal momenta. As seen above, for a range of arbitrary initial values, this requirement is met.

- Using the final momenta of the particles, the collision rate of the interaction was derived.
- This rate is dependent on all the parameters mentioned earlier.

$$\Gamma = \frac{CT_L^5}{2^{11}\pi^7 m_X^4}\int \frac{k'(p^{*\prime})^4}{\tilde{k}'}\left[\frac{3}{2} - \frac{1}{2}\cos\theta_{CM}\right]\frac{\sin\phi_{CM}\sin^2\theta_{CM}}{\tilde{p}_{x,0}h(\tilde{p}_{x,0}^2)}\left[2\frac{pp^*}{m_X^2}\cos\theta_i + \gamma\gamma_X^*(1+\gamma_X^*\frac{p^{*2}}{m_X^2}\cos 2\theta_i)\right] \times \left[1 - g(\tilde{k}\right] g(k)\, dk'\, d(\cos\theta_i)\, d\phi_{CM}\, d\theta_{CM}$$

- Using the final momenta of the particles, the collision rate of the interaction was derived.

$$\Gamma = 1.23373\times10^{-10}\gamma\frac{m_X}{T_L}$$

- The above value is used as it is in terms of the momentum transfer rate, γ. Once we have calculated γ, we can also calculate the time of decoupling between the particles and the resultant final velocity distribution of dark matter.

CONCLUSION

Compared to previous work by Charlie Mace in his thesis *Simulating the Thermal Evolution of Dark Matter During an Early Matter-Dominated Era*, there is a discrepancy of many orders of magnitude[1]. Throughout the process of this work, there were a few inconsistencies. Most of these inconsistencies were different constant factors, however, there was a large disparity in the calculation of the collision operator integral. Further comparison to other studies of similar work must be done to converge on a final value for the collision rate of these interactions.

[1]Mace, Charlie. Simulating the Thermal Evolution of Dark Matter During an Early Matter-Dominated Era. Chapel Hill: Department of Physics and Astronomy, 2020.