

In-Medium Similarity Renormalization Group

The IMSRG is an ab-initio method used to calculate the ground state energy of a many-body system. It works as follows:

- Choose a reference state (“in-medium”) for your system
- Evolve the Hamiltonian via a unitary transformation $U(s)$ (“similarity”), where s is the flow parameter that parametrizes the transformation
- Continue evolving until we decouple the reference state from excited states (“renormalization group”)

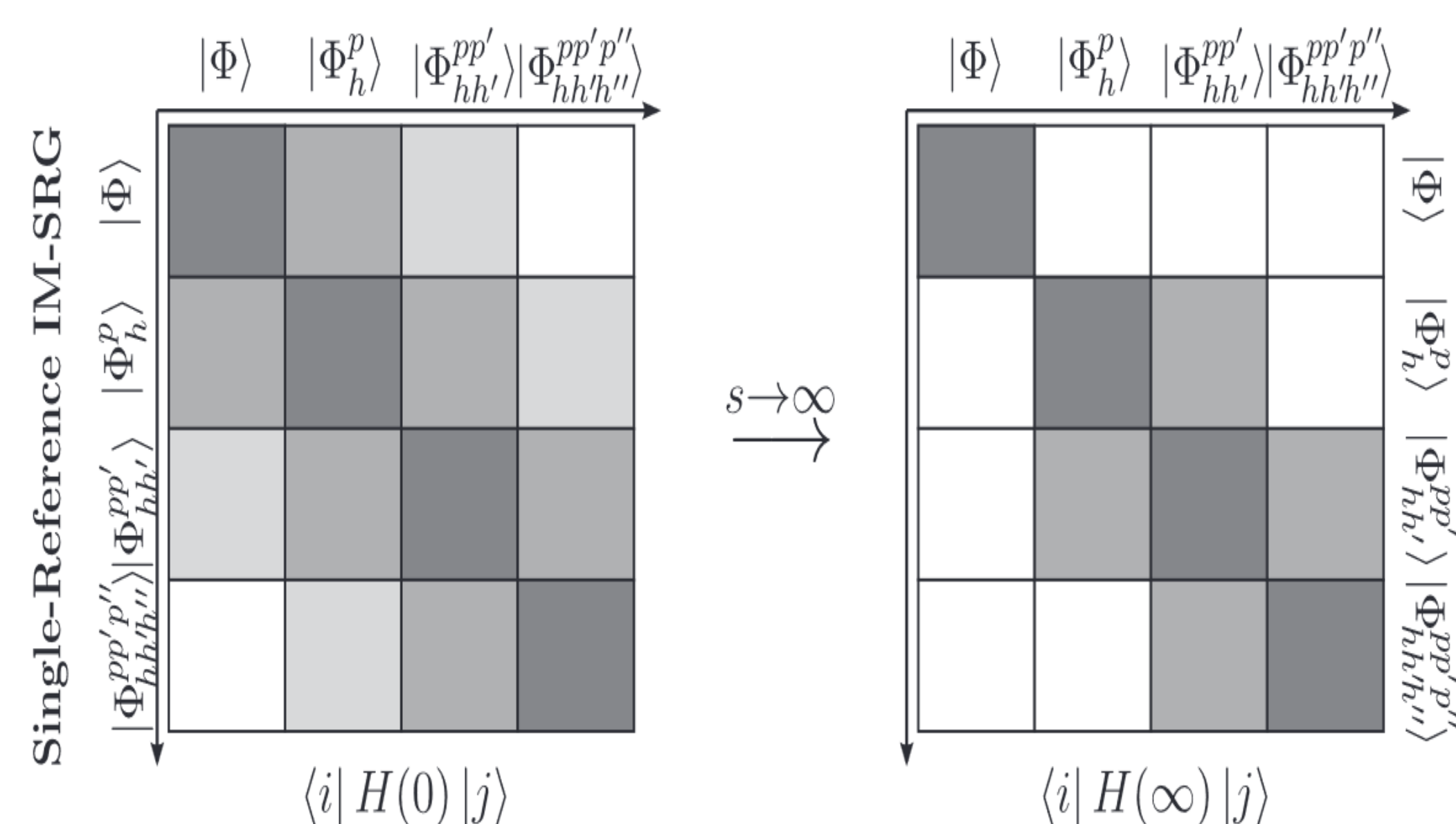


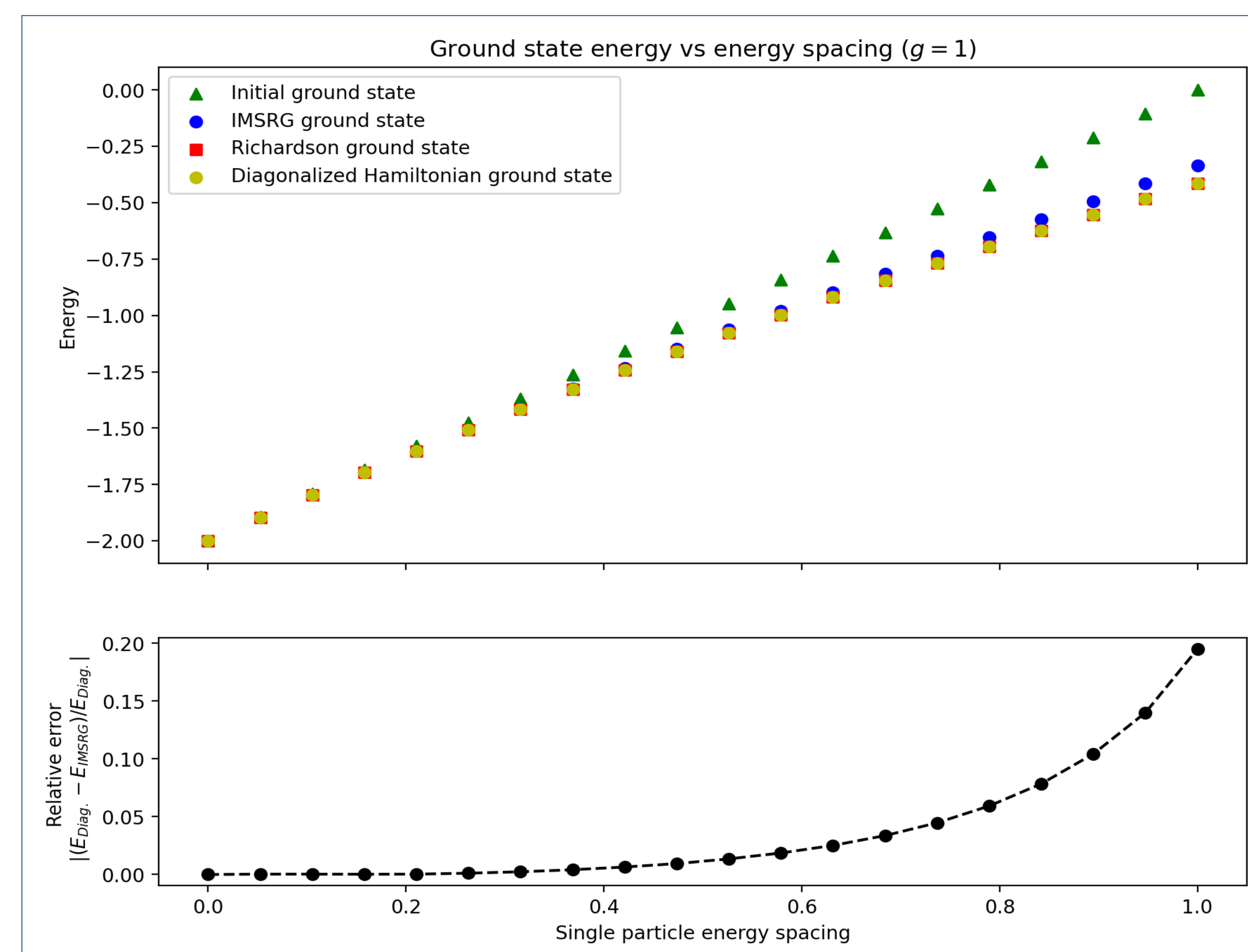
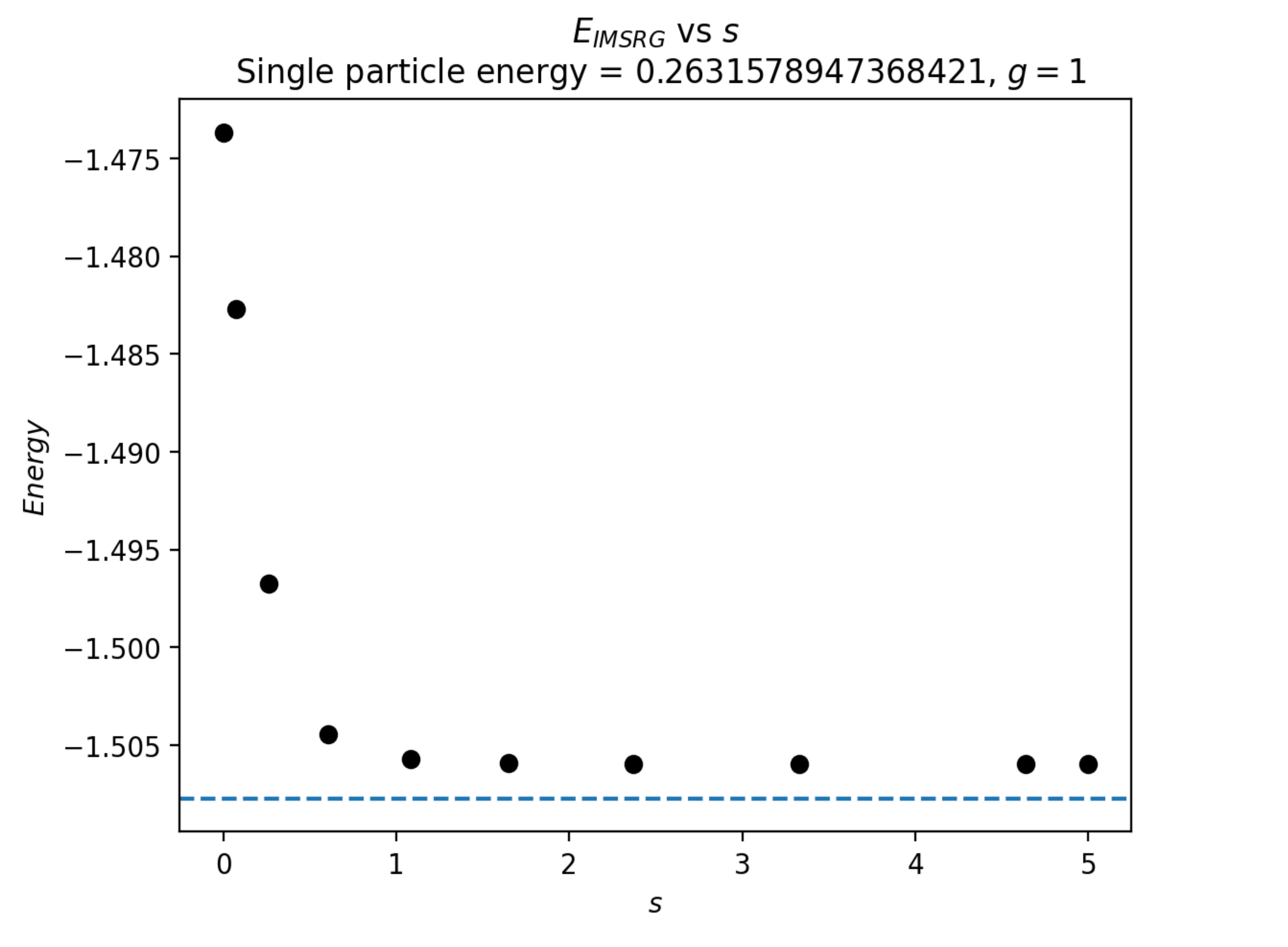
Figure taken from reference (1)

Computationally, we evolve H via $U(s)$ by solving the differential equation

$$\frac{dH}{ds} = [H, \eta(s)]$$

where $\eta(s)$ is the generator of $U(s)$.

Issues with the IMSRG are that it is very computationally expensive. Our solution to this is to use symmetries of our system to perform analytic calculations of certain matrix elements, reducing the computation time.



References

1. Hergert, H. (2016). In-medium similarity renormalization group for closed and open-shell nuclei. *Physica Scripta*, 92(2), 023002. INSPIRE. <https://doi.org/10.1088/1402-4896/92/2/023002>

Nuclear Pairing Hamiltonian

We consider a system of many nucleons in a single angular momentum j -shell which have strong pairing correlations. The Hamiltonian for this system is given by

$$H = \sum_m \epsilon_m a_m^\dagger a_m + \frac{1}{4} \sum_{m,m'} g_{m,m'} (-1)^{-m-m'} a_m^\dagger a_{-m}^\dagger a_{-m'} a_{m'}$$

where g is the strength of the pairing interaction, and ϵ_m are the single particle energies.

The 2-body term has $su(2)$ symmetry under seniority (number of unpaired nucleons), and thus has eigenstates given by states of definite seniority. Thus for $g \gg \epsilon_m$ we can approximate the energy eigenstates as those of definite seniority.

Computational Scheme

To find the ground state energy of H using the IMSRG, we used the seniority 0 state as our reference state due to it being a close approximation to the true ground state. The $su(2)$ structure allowed us to analytically calculate relevant matrix elements using the Wigner-Eckart theorem, improving the computational efficiency.

Further, we used the Brillouin generator for our calculations, and took s to range from 0 to 5, as this was seen to be sufficient for the IMSRG to converge.

Future Work

The IMSRG doesn't fully decouple the reference state we chose. To improve the result for the ground state energy, we could diagonalize the output of the IMSRG in a reduced subspace